# Fallen Angel Investment Strategies

## A research study

at

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## Abstract

Fallen angels, defined as the stocks within an index that show the worst performance in a certain timeframe, are hypothesized to be slightly undervalued in highly correlated markets. That is, through their correlation with the Index, the fallen angels plausibly revert to their historical mean, generating a small outperformance. In this mathematical analysis, we study the possibilities of exploiting this mean reversion by formulating Fallen Angel investment strategies in a correlated market. The study is based on ten years of historical data of a leading index in the property sector, the GPR 250 Property Securities Index. The results show respectable annual performances for several fallen angel investment strategies. Subsequently, an extensive analysis of the properties of such strategies reveals the correlation between the volatility of the index and the risk of the strategy, and the stable, outperforming basis of fallen angel strategies.

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## 1.0 Introduction

“If you don’t follow the stock market you are missing some amazing drama” – Mark Cuban

Many investors would undoubtedly agree with the billionaire internet entrepreneur Mark Cuban. According to Cuban, the stock market is one of the most dynamical market places in the world. On no other market are changes in prices so seemingly unpredictable. On no other market are the stakes so high. The Dutch worldwide trading enterprise VOC issued the very first shares back in the $17^{th}$ century, and so gained the massive starting capital necessary to make the expensive voyages around the world. Shareholders were rewarded for their trust in de company by receiving part of the VOC’s profit – dividend. Other companies followed the set example. Nowadays, every self-respecting firm of decent size issues shares. Still, shareholders often receive part of the company’s profit in the form of a dividend in return for their trust. Every second, immense quantities of these shares change hands on digital stock markets. The purchase of thousands of shares of a company of their interest is just one click away. On such a dynamical, lively market, different developments quickly follow one another. Enormous highs alternate great lows, both for the listed companies, as for the active traders. The former group’s main motive for issuing shares is the acquisition of capital. The latter group independently attempts to make a profit by trading on the stock market. Retirements funds, governments, banks and private stock investors, they all try to extend their fortune with wise investments. Generally, such investments are preceded by teams of analysts who have carefully researched specific parts of the market for a long time. Such analyses stretch from portfolio selection analysis to studies to dividend; from analysis of a complete index with stocks to inquiry of one specific company. With this research, yet another of such analyses will add its share of information that is meant to eventually lead to the most profitable of investments: fallen angels investment strategies; a research study.

The hypothesis that precedes this research is that many fallen shares are temporarily undervalued. That is, fallen stocks are generally expected to revert to their historical mean – particularly in correlated markets. This hypothesis opens the door to a study to the possibilities of profitably exploiting this mean reversion through fallen angel investment strategies in correlated markets. Precisely this will be the approach of this research.

## 2.0 Research projective

The overall projective of this research is to reveal the possibilities of fallen angel investment strategies in correlated markets. We wish to evaluate the historical performances of a variety of fallen angel strategies, governed by two parameters: the number of selected fallen angels and the holding period in days. The variety of strategies naturally distinguish themselves in performance through time. However, we also wish to evaluate other characteristics of these strategies in order to expose the origin of the strategy's performance. These characteristics include volatility of the strategy through time, dependence on few large peaks in individual performances, and dependence on few volatile stocks. The analysis of such characteristics discloses to what extent the historic performances have any predictive value for the future.

### *2.1 Fallen Angels in this research report*

In finance, the concept 'fallen angels' is alternately used to denote different things. On the one hand, fallen angels are defined as bonds that are downgraded from investment-grade to speculative-grade ratings by some of the well-known rating agencies. The process of downgrading a fund by a rating agency is rather laborious process; it may take days, even weeks from the fall of a stock. This does not allow any transactions of fallen angels to be made on the short term. However, the short term is just the ground on which we expect a fallen angel strategy to be able to generate a profit. Therefore, we will refrain from any further references to rating agencies.

Another passable definition describes fallen angels as stocks that have fallen steeply from their former high. We use a more general derivative of this definition in this study: the funds within an Index that show the worst performance in a certain period. This way, we allow a short term focused fallen angel investment strategy to flourish.

Fallen Angels: A number of funds within an Index that show the worst performance in a certain period

### *2.2 Coherence in the Property Investment Sector*

Mean reversion, the concept in which this research originates, is expected to be more applicable to correlated markets. If there is a high coherence within the index, the companies are less likely to behave as independent entities, and more likely to be affected by the trends of the index as a whole. Through the great coherence within the sector, stocks whose recent performance greatly differs from their historical averages are more likely to revert back to the mean. The more likely fallen stocks are to revert to their historical mean, the better the performance of a fallen angel investment strategy. In this light, we wish to perform the strategy in a highly correlated index.

One of the economic sectors that has a great coherence is the property investment sector.

For companies operating in the property industry, the most important fundamental drivers that govern their performance are all macro-economic phenomena: inflation (rent changes) and interest rates (funding). Contrary to many other economic sectors, all entities in the property investment sector are predominantly dependent on just these two macro-economic factors, which makes the property investment sector an extremely static sector. Therefore property investment companies plausibly correlate to a great extent.

Moreover, bankruptcies are much less abundant within the property sector than in many other industries. The main reason for this is that, the very core of their business, their property, gives property investment companies collateral in order to avoid bankruptcies. Evidently a fallen angel ending up bankrupt completely annihilates the performance of a fallen angel strategy, so a decreased likelihood of bankruptcy greatly favors the property investment sector as a playground for fallen angel investment strategies. Finally, unlike other correlated sectors – the airline industry for example – the property investment sector is a rather sizable industry, which allows a substantial coherent index.

For these reasons, we choose the property sector to act as our playground to perform fallen angel investment strategies.

## 2.0 Dataset

The previous discussion suggests the utilization of a dataset that contains a large amount of daily prices and dividends of a portfolio of the biggest, most volatile property investment companies of the world. Such an index exists in the form of the GPR 250 Global Property Securities Index.

### *2.1 Global Property Research*

The GPR 250 Global Property Securities Index is a composition of the 250 global listed property securities with the highest monthly trading volume. GPR uses a variety of inclusion rules to ensure a volatile index that forms a sustainable representation of the global property market. Primarily, only property investment companies and hybrid property companies may be included in the index. Property investment companies are defined as companies for which at least 75% of operational turnover is derived from investment activities. For hybrid property companies, at least 75% of operational turnover must be derived from investment and development activities combined, of which a third (25% of total) must be derived from investment activities. Secondly, for a company to be candidated for inclusion, the free float market capitalization must be over 50 million USD for two consecutive months, with a free float percentage of at least 15%. Moreover, newly listed companies (IPOs) must rank among the top 150 to be included in the GPR 250. All companies that meet these requirements are eligible for inclusion in the GPR-250 index. Of these candidates, a maximum of 250 companies with the highest monthly trading volume will be included.

These inclusion rules make the GPR 250 Index an ideal playground for an investigation of fallen angel investment strategies. The market capitalization and liquidity criteria guarantee the possibility of trading every stock in the dataset at any desired time up to any reasonable amount. Moreover, with available daily data of constituents of the GPR 250 Index reaching back to 3 January 2000, the database of the GPR 250 Index is rather extensive. Since, ceteris paribus, the larger the dataset, the larger the statistical significance of the results, in light of statistical significance too, the GPR 250 Index forms a great dataset for the purpose of this research.

From all constituents, a capitalization-weighted index based on the market value of these shares is calculated (Global Property Research, Amsterdam). This index is used to evaluate the strategies' performances against.

$$W_{i,t} = \frac{C_{i,t}}{\sum_{i=1}^{N_t} C_{i,t}} \tag{1}$$

$$I_{index,t} = I_{index,t-1}\left(1 + \sum_{i=1}^{N_t} W_{i,t} \cdot r_{i,t}\right) \tag{2}$$

$W_{i,t}$ Weight of company $i$ at time t

$C_{i,t}$ Free float market capitalization of company $i$ at time t

$N_t$ Number of companies that meet the inclusion criteria at time t

$I_{index,t}$ Index value at time t. Base value is 100 at 3 January 2000.

$r_{i,t}$ Total return of company $i$ at time $t$. Total return will be explained later.

### *2.2 Dataset*

In practice, the dataset consists of three different types of information: the date, company information, and share information for shares of all constituents of the GPR 250. Recall that the exact composition of the GPR 250 may change over time, so that our dataset will consist of more than 250 stocks. However, for all stocks, only the data from the period that they were included in the GPR 250 will be part of our dataset. The available data on daily basis reaches back to 3 January 2000. With the stock market open 5 days a week, this comes down to over 2700 dates. The company information includes the commercial name, Bloomberg code, continent code, country code, and a constituent ID, a number referring to the specific company, remaining constant even if the commercial name of the company changes. Finally, the share information includes daily closing price, dividend, and a daily adjustment factor, a multiplier that corrects for changes in the intrinsic value of the stock not having return implications. This avoids complications in case of a change in the number of issued shares.

## 3.0 Correlations

As discussed before, the property sector is hypothesized to show great coherence in general. In other words, the constituents of the GPR 250 Global Property Securities Index, a sustainable representation of the global property market, are expected to be greatly correlated. Let us put this theory to the test.

### *3.1 Pearson correlation coefficient between continent indices*

One of the most well-known measures of linear dependence between two quantities in science is the Pearson's product-moment correlation coefficient (PMCC), often denoted by $r$. Giving a value between -1 and +1, the PMCC is a measure of the correlation between two lists of variables. This correlation coefficient, developed by Karl Pearson, is defined as the sum of the products of the standard scores of the two measures divided by the degrees of freedom. The result obtained is equivalent to the covariance of the two variables divided by the product of their standard deviations.

$$r_{xy} = \frac{\text{cov(X, Y)}}{\sigma_X \sigma_Y} = \frac{\overline{XY} - \bar{X} \cdot \bar{Y}}{\sigma_X \sigma_Y} \quad (3)$$

$r_{X,Y}$ Pearson's product-moment correlation coefficient.

$\sigma_x$ Standard deviation in variable X as discussed before

$\sigma_y$ Standard deviation in variable Y as discussed before

$\overline{XY}$ Expected (average) value of the product XY

$\bar{X}$ Expected (average) value of the variable X

$\bar{Y}$ Expected (average) value of the variable Y

The Pearson correlation is defined only if both stand deviations are finite and non-zero. As said, the correlation will take values between -1 and 1. In this respect, a value of 1 indicates a perfect positive linear relationship, a value of -1 a perfect negative linear relationship. Although the PMCC is a very reliable measure, it does not take into account the size of the fluctuations. As long as the fluctuations have the same directions and are proportional to one another, the correlation is not affected.

Figure 3.1 shows the values for the correlations between the continent indices and the entire GPR 250 and the correlations between the continents themselves. Naturally, the correlation between the same indices is +1. It is not surprising that the correlation coefficients between the continents and the GPR 250 Index are as extremely high as they are. The GPR 250 Index is a coherence of the indices divided by continents, so particularly for the bigger contributing continents, you would expect high correlations. However, the correlation between the different continent indices is definitely not self-evident. The fact that the correlation between the biggest property investment companies in Europe and America is 0.969 shows how insanely correlated the property sector is. In particular, as the GPR 250 Index is a capitalization-weighted index, companies with a relatively large market capitalization are highly correlated.

| | EUROPE | AMERICA | ASIA | AUSTRALIA | GPR250 |
|---|---|---|---|---|---|
| EUROPE | 1. | 0.969077 | 0.91919 | 0.943853 | 0.983829 |
| AMERICA | 0.969077 | 1. | 0.926842 | 0.927728 | 0.990278 |
| ASIA | 0.91919 | 0.926842 | 1. | 0.868783 | 0.959508 |
| AUSTRALIA | 0.943853 | 0.927728 | 0.868783 | 1. | 0.946582 |
| GPR250 | 0.983829 | 0.990278 | 0.959508 | 0.946582 | 1. |

*Figure 3.1: Table showing the Pearson's correlation coefficients between the continent indices.*

### *3.2 Correlation coefficients between constituents*

In the book *An introduction to Econophysics; Correlations and Complexity in Finance,* Mantegna and Stanley suggest another way to compute correlations coefficients; in particular the correlations between individual stocks. "One way to detect similarities and differences in the synchronous time evolution of a pair of stocks is to study the correlation coefficient $\rho_{ij}$ between the daily logarithmic changes in price of two stocks $i$ and $j$." (Mantegna and Stanley) If we define for stock $i$

$$S_i \equiv \ln P_{i,t} - \log P_{i,t-1} \quad (4)$$

then

$$\rho_{ij} = \frac{\langle S_i S_j \rangle - \langle S_i \rangle \langle S_j \rangle}{\sqrt{\langle {S_i}^2 - \langle S_i \rangle^2 \rangle \langle {S_j}^2 - \langle S_j \rangle^2 \rangle}} \quad (5)$$

$S_i$ Daily change of the logarithm of the price of stock $i$

$P_{i,t}$ Daily closing price of stock $i$ at time $t$

The angular brackets indicate a time average over all the trading days within the investigated time period. The correlation coefficient $\rho_{ij}$ too can take values ranging from -1 to 1, where a value of 1 indicates completely correlated changes in stock price, and a value of -1 indicates completely anticorrelated changes in stock price. Typically, these correlation coefficients are considerably closer to zero than Pearson's correlation coefficients.

Let us investigate the coherence of the property sector by finding the correlation coefficients between constituents of the GPR 250 Index. To ensure continuity of coefficient calculations through varying compositions of the index, we will disregard companies that were not included in the index throughout the entire investigated time period. A total number of 105 property companies have been continuously included in the GPR 250 Index throughout the entire length of our dataset, from January 2000 till 28 May 2010. For the set of these 105 stocks, there are (105 x 104)/2 = 5460 different $\rho_{ij}$. The strongest correlation is observed between Boston Properties Inc and Vornado Realty Trust at 0.883. The anticorrelations are considerably weaker, with the strongest observed anticorrelation at -0.120 between Equity Residential Props Trust and Liberty Property Trust. Figure 3.2 summarizes the distribution of correlation coefficients. Clearly, the correlation coefficients are far from normally distributed. The most remarkable aspect of this graph is the small peak around 0.7. A $\rho_{ij}$ of 0.7 is enormous. Apparently, a great number of property investment companies are highly correlated. As a comparison, the largest recorded $\rho_{ij}$ between 1990 and 1994 in the S&P 500, a market-capitalization weighted index with the 500 biggest companies in the United States, is between Homestake Mining and Placer Dome, Inc. for which $\rho_{ij} = 0.82$. (Mantegna and Stanley) The strongest anticorrelation in the same dataset is between Barrick Gold and Nynes Corporation at $\rho_{ij} = -0.30$.

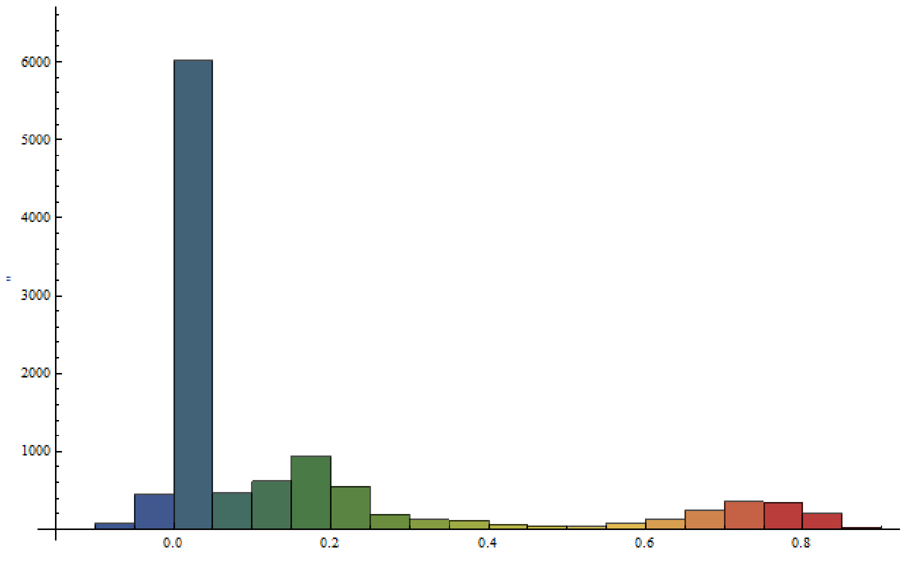


Figure 3.2; Distribution of correlation coefficients between the 105 companies that were continuously included in the GPR 250 Index in the past ten years.

The average $\rho_{ij}$ for these 105 most stable property investment companies with a relatively large market capitalization lies at 0.289. In comparison, between 1990 and 1994, the average correlation coefficient in the S&P 500 was about 0.18. (Mantegna and Stanley) These results all suggest that the property sector in general and the GPR 250 Global Property Securities Index in particular correlate to a much greater extent than more general indices as the S&P 500.

## 4.0 Research Procedure

Now that the research objective and the anticipated dataset are carefully formulated, we are yet to establish the link between the dataset and the expressed objectives: the research procedure. The first step will be to manipulate the rough dataset in such a way, that it can be used efficiently for the purpose of our research. Through this process, the complete dataset with all the information as stated above needs to be converged in one huge matrix with dimension {number of dates, number of stocks} and indexed total returns as column vectors.

### *4.1 Market shares, merges, and adjustment factors.*

The first step in modifying the dataset is to account for changes in the number of available shares. Both the prices and the dividends for all stocks need to be adjusted for changes in the intrinsic value of the stock, such that there are no implications for the total return. In case of a stock split, a stock merge, a stock dividend, a poison pill, a rights issue or a bonus issue, the number of available shares for that stock changes. To ensure continuity of return calculations, one should adapt the prices and dividends to the change in available market shares. This is done through the implementation of an adjustment factor

### *4.2 Price return, Dividend return and Total return*

We calculate the concepts price return, dividend return, and total return as follows.

$$pr_{i,t} = \frac{P_{i,t}}{P_{i,t-1}} - 1 \qquad dr_{i,t} = \frac{D_{i,t}}{P_{i,t-1}} \qquad r_{i,t} = pr_{i,t} + dr_{i,t} = \frac{P_{i,t} + D_{i,t}}{P_{i,t-1}} - 1 \tag{9}$$

$P_{i,t}$ Closing price of one share of company $i$ at time $t$

$P_{i,t-1}$ Closing price of one share of company $i$ at time $t-1$

$pr_{i,t}$ Price Return of company $i$ at time $t$

$D_i$ Dividend paid to shareholders per share by company $i$ at time t

$dr_{i,t}$ Dividend Return of company $i$ at time $t$

$r_{i,t}$ Total Return of company $i$ at time $t$

### *4.3 Indexed Total return*

Throughout this research, total return calculations will not be limited to daily returns. However, since the total return does not behave linearly, subsequent total returns cannot be added.

In light of this, it is useful to index the total returns for all stocks. We choose the base date to be at the start of the dataset, 3 January 2000, and the base value to be 100.

$$I_{i,t} = I_{i,t-1}(1 + r_{i,t}) \tag{10}$$

$I_{i,t}$ Indexed total return of company i at time $t + 1$

$I_{i,1}$ Base value at 3 January 2000 $= 100$

$r_{i,t}$ Total return of company i at time t

### *4.4 Data handling*

The data that results from the discussion above is to be manipulated such that it can easily be accessed and be used efficiently in our research. We choose to construct one large matrix with the dimension {number of dates, number of stocks}. The indexed total returns per stock will appear as column vectors in this matrix. This matrix is imported in Mathematica®. Through this matrix, total returns of several stocks over any period may be compared in the most efficient way.

| | 117 | 120 | 121 | 124 | 125 | 126 | 127 |
|---|---|---|---|---|---|---|---|
| 3-Jan-00 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 4-Jan-00 | 100.478 | 100 | 98.9712 | 98.0769 | 100 | 99.7817 | 100.586 |
| 5-Jan-00 | 100 | 99.262 | 96.9136 | 96.1538 | 100 | 97.5983 | 101.171 |
| 6-Jan-00 | 100.159 | 97.786 | 96.2963 | 97.5962 | 100 | 97.5983 | 105.124 |
| 7-Jan-00 | 101.592 | 97.786 | 96.2963 | 97.1154 | 100 | 98.0349 | 113.324 |
| 10-Jan-00 | 103.822 | 97.786 | 104.733 | 97.1154 | 100 | 98.4716 | 117.862 |
| 11-Jan-00 | 103.981 | 97.786 | 106.79 | 96.7308 | 100 | 100 | 116.398 |
| 12-Jan-00 | 103.822 | 97.786 | 106.79 | 97.1154 | 100 | 103.712 | 116.105 |
| 13-Jan-00 | 103.185 | 97.786 | 106.79 | 97.1154 | 100 | 106.987 | 113.616 |

Figure 4.1: Sample of the manipulated, polished dataset that is imported into Mathematica®. The numbers 117 through 127 are constituent IDs.

With the polished dataset imported in Mathematica®, the first visualizations of the data can be made. Below follow graphical illustrations of the GPR indices per continent.

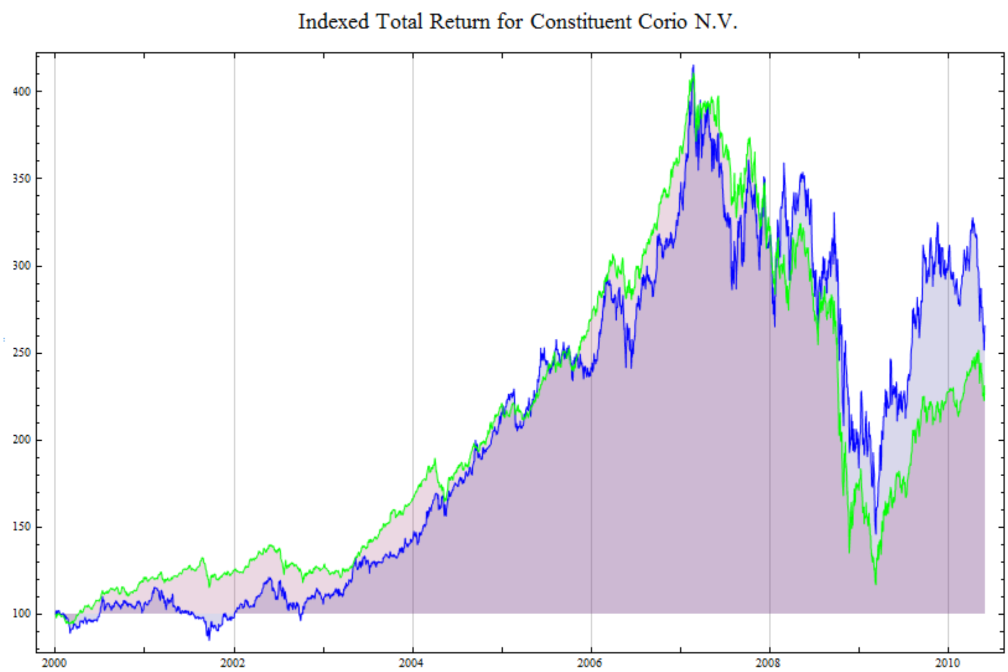


Figure 4.2: Indexed Total return of Corio N.V. (blue) and the GPR 250 Index (green).

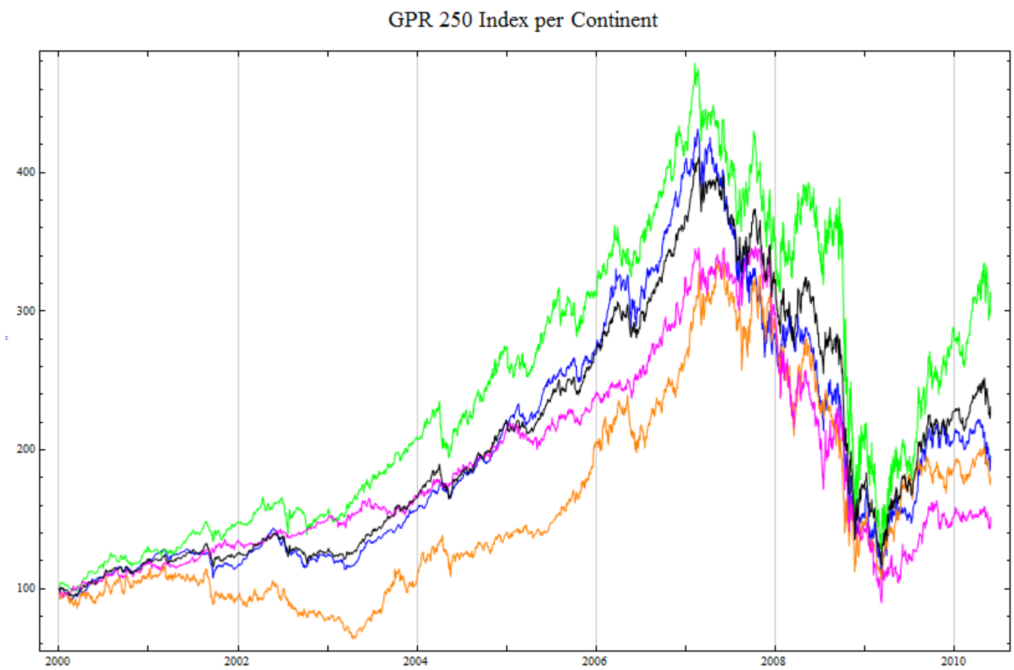


Figure 4.3: Graph of total return of the entire GPR 250 Index (black) and the separate continent indices: Americas (green), Europe (blue), Asia (orange) and Australia (magenta).

## 5.0 Dynamical analysis of Fallen Angels

The strategy we wish to formulate is based on the principles correlation, coherence, and mean reversion. We wish to use these concepts to our advantage by continuously selecting fallen angels in a portfolio. Consequently, the fallen angel portfolios constantly change. At each selection point, the former portfolio is replaced by new fallen angels. In this definition of fallen angels we have formulated, both the number of worst-performing shares and the selection period are subject to change. These parameters govern the different fallen angel strategies. Figure 5.1 shows a graphical illustration of a fallen angel investment strategy. The strategy consists of two steps that endlessly repeat themselves through the entire period: identifying and the fallen angels in a selection period on the one hand, selling this portfolio after the holding period on the other hand. To ensure continuity of the strategy with equal amounts of stocks in the fallen angel portfolio, the holding period is chosen to be equal to the selection period.

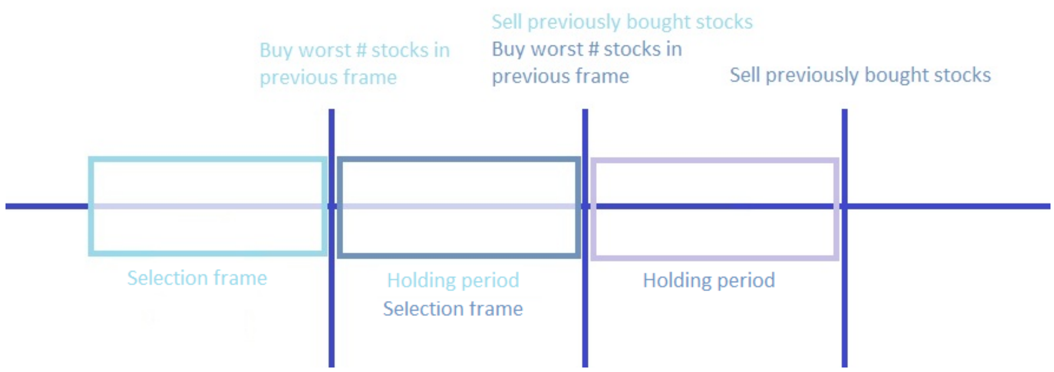


Figure 5.1: Graphical illustration of a Fallen Angel Investment strategy. After every period, one selects the *n* worst stocks and holds them for the same period. The parameters that govern such a strategy are the number of fallen angels and the length of the holding period/selection frame.

### *5.1 Identifying Fallen Angels in selection frame*

First, for any selection frame, stationary point, and number of fallen angels we must identify the corresponding fallen angels. That is, we calculate the return of all stocks in a given selection frame, and select the *n* smallest values.

$$r_{i,[s-t,s]} = \frac{I_{i,s}}{I_{i,s-t}} - 1 \tag{11}$$

$r_{i,[s-t,s]}$ Total return of company i in selection period

$s$ Selection point in days after 3-jan-2000

$t$ Selection frame in days

$I_{i,s}$ Indexed total return of company i at time s

From these total returns, we select the *n* smallest values. These companies are the *n* fallen angels in the given selection period, so they are included in the fallen angel portfolio at the selection point *s*.

### *5.2 Compare performance of Fallen Angels with index.*

Next, we need to analyze the performance of these Fallen Angels in the period after the selection. The absolute value of the performance is not in our interest; instead, we need the relative performance with respect to some bigger index of stocks of which the fallen angels are part of, the GPR 250 Index. That is, we average all the total returns of the fallen angels, and subtract from it the total return of the index in that same period.

$$\mathrm{pf}_{\mathrm{fa},[\mathrm{s},\mathrm{s}+\mathrm{h}]} = \sum_{i=1}^{n} \left[ \frac{1}{n} \frac{I_{i,s+h}}{I_{i,s}} \right] - \frac{I_{index,s+h}}{I_{index,s}} \tag{12}$$

$\mathrm{pf}_{\mathrm{fa},[\mathrm{s},\mathrm{s}+\mathrm{h}]}$ Performance of fallen angels in holding period

$n$ Number of fallen angels

$h$ Holding period in days

$I_{i,s}$ Indexed total return of company i at selection point $s$

$I_{index,s}$ Index value of GPR 250 Index at time s

The result is the relative performance of the selected Fallen Angels in a period after the selection point, the holding period.

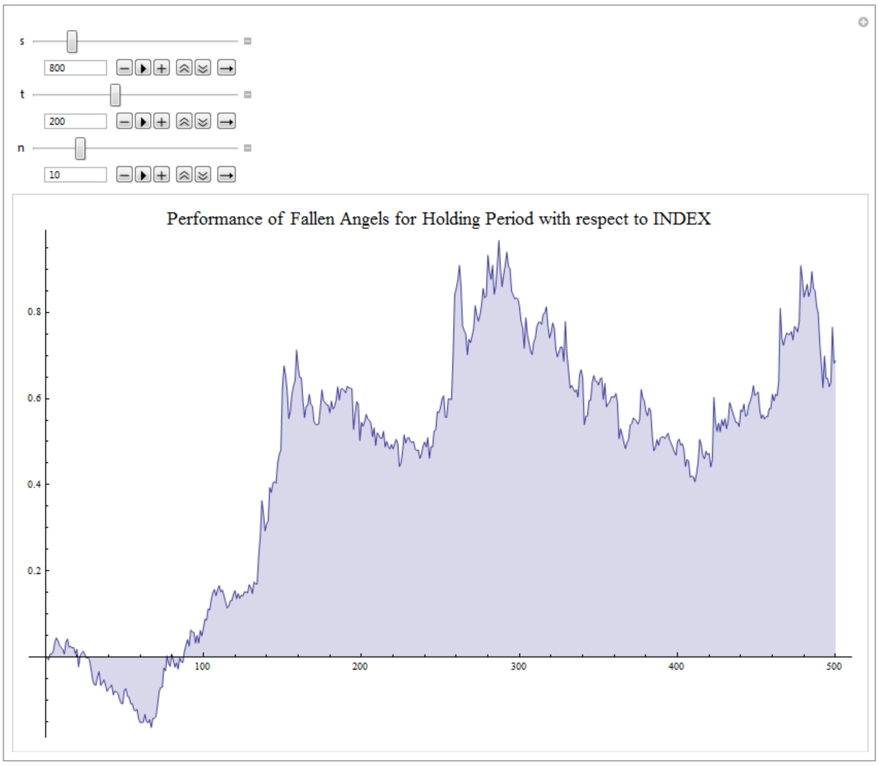


Figure 5.2: Example plot of stationary analysis with *n = 10, t = 200, and s = 800.* That is, at the origin of the plot, 800 days from 3 January 2000, we select the 10 worst-performing stocks in the past 200 days. The x-axis gives the holding period of the selected Fallen Angels. The y-axis gives the performance of this portfolio with respect to the GPR-250 Index in percentages. A value of 0.8 means that the fallen angels outperform the GPR-250 Index by 0.8 percentage point.

## *5.3 Total return of Fallen Angel investment strategy*

The total return that you would have acquired with this strategy is the combination of individual returns of the selected fallen angels in each holding period. In combining these separate returns, one needs to recall that they cannot be added linearly. Instead, the separate returns should be combined as follows.

$$\mathrm{pf}_{\mathrm{strategy},[1,\mathrm{T}]} = \prod_{s=h+1}^{T}\left[\sum_{i=1}^{n}\left[\frac{1}{n}\,\frac{I_{i,s+h}}{I_{i,s}}\right] - \frac{I_{index,s+h}}{I_{index,s}} + 1\right] - 1 = \prod_{s=h+1}^{T}\left[\mathrm{pf}_{\mathrm{fa},[\mathrm{s,s+h}]} + 1\right] - 1 \qquad (13)$$

$\mathrm{pf}_{\mathrm{strategy},[1,\mathrm{T}]}$ Total Performance of Fallen Angel Strategy in period [1,T]

$T$ End date of strategy = 28 May 2010 minus one holding period $= 2715 - h$

## *5.4 Parameters*

Each strategy is defined by two variable parameters. The number of selected fallen angels *n*, and the selection/holding period *h*. As said, we choose the selection period to be equal to the holding period. Initially, we let these parameters vary in the following way.

Number of fallen angels *n*: {1,2,3,4,5,6,7,8,9,10,13,15,20,25}
Holding/Selection period *h* (days): {1,2,3,4,5,10,25,50,100}

Regarding these parameters, we may expect the following. Any result of the fallen angel strategy will be magnified as the fallen angel portfolio includes less stocks. The more fallen angels are selected, the better your portfolio is representative for the index. So, as more stocks are included in the fallen angel portfolios, the performance of the strategy plausibly converges to the performance of the index. The standard deviation, on the other hand, will probably travel the reverse path.

## 6.0 Results

Our first and foremost concern is the historic results of the fallen angel strategies in the past ten years. That is, the returns that would have been obtained if the named strategies were carried out in the GPR 250 Index over the past ten years. A straightforward exercise transforms our strategy performance into a similar table with average annual performances. The length of our dataset is 2716 days. With an average of 261.1 working days per year in this dataset, this comes down to a period of 10.40 years. The average annual return is calculated by taking the 10.40$^{th}$ root of the total performance.

$$\text{Average annual performance} = \sqrt[10.40]{\text{Total Peformance of Strategy}}$$

This operation clearly does not alter the optimal strategy. It merely gives the performance of the strategy in a figure that lets itself compare with other investment strategies more easily. Figure 6.1 shows the historic average annual performances of a variety of fallen angel strategies.

Number of Fallen Angels in Portfolio

| Holding period | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1 | 84.8 | 53.5 | 35.3 | 22.0 | 10.5 | 4.18 | 3.68 |
| 2 | 17.5 | 18.5 | 24.3 | 21.3 | 19.7 | 14.3 | 10.1 |
| 3 | 106. | 54.5 | 56.7 | 48.9 | 42.1 | 40.7 | 41.3 |
| 4 | 23.3 | 57.3 | 43.0 | 31.8 | 33.4 | 29.2 | 26.3 |
| 5 | 28.2 | 39.3 | 42.9 | 49.2 | 46.8 | 42.7 | 38.2 |
| 10 | 38.6 | 41.1 | 31.1 | 28.7 | 24.5 | 20.9 | 22.3 |
| 25 | 15.5 | 23.0 | 22.2 | 23.4 | 19.5 | 20.7 | 18.1 |
| 50 | 6.66 | 21.7 | 12.5 | 12.3 | 2.83 | 2.72 | 2.50 |
| 100 | -16.3 | -0.756 | -4.68 | -0.282 | -3.21 | -3.06 | -1.69 |

Figure 6.1: Table showing the average annual performance per strategy with respect to the GPR 250 Index in percentages. The rows are holding periods in days, the columns are the number of selected fallen angels. Further extensions of the parameter *n* are left out here, as the results showed that our hypothesis was correct: as the number of selected fallen angels increases, the total return of the strategy converges to the total return of the index. Therefore, any portfolios of fallen angels larger than 7 stocks are of no interest at this point.

The results show an enormous return for strategies with a short holding period and a small portfolio of fallen angels. There is a clear trend of rising return to the left top corner, with strategies with holding periods of 2 or 3 days as exceptions. Moreover, fallen angel strategies with holding periods of 100 days and longer are eligible to give negative returns. Apparently, a fund that has underperformed for such a long period is no longer undervalued and has little chance to revert to its historical mean. That is, these longer term underperformances are preceded by structural negative changes, rather than short term deviations.

## 7.0 Discussion

At first sight, with annual performances of up to 105%, fallen angel investment strategies in correlated markets as the GPR 250 seem to generate huge outperformances. However, there are some catches that need to be seriously considered, before one projects the historic performances on the future.

### *7.1 Intraday liquidity*

Daily closing prices have been picked as the source of information throughout this research. Therefore we assumed the closing price to be representative for the price at which that stock was traded throughout the day. We have not corrected for intraday price changes. In volatile times in particular, this may bias the results of the research. According to the 1988 study *A theory of Intraday Patterns: Volume and Price Variability* by Admati and Pfleiderer, trading volume tends to be concentrated in particular time periods within the trading day. Moreover, through a correlation between trading volume and return variability, it is argued that returns are more variable in similar intraday time periods. The shape of the changes in return variability throughout a day is dependent, among others, on the division of traders in nondiscretionary liquidity traders[1] and discretionary liquidity traders[2], and on the rate of arrival of information in that market. Typically, the return volatility is larger at the start and at the end of the trading day. On top, the intraday liquidity within an index is naturally proportional to the overall volatility of the Index. For this reason, the closing prices are generally not completely representative for the trading prices throughout the day in the volatile GPR 250 Index. This may lead to slightly biased results, particularly in volatile times.

### *7.2 Costs per trade*

Throughout this report, the costs have been approximated at 15 basis points (0.15%) per transaction. For a complete renewal of the Fallen Angel portfolio, two transactions are needed, which brings the costs per trade at 30 basis points (0.30%). That is, we have implemented the costs per trade by subtracting 0.30% from the individual performances of the fallen angel portfolios.

This is a perfectly fine approximation for Asia and Europe. However, the system of transaction fees in America is structured subtly different. As the fees are paid in a fixed amount of dollars per transaction, the relative costs per trade differ greatly with the price of the stock. For this reason, the cost approximation of 0.30% may give biased results for American constituent stocks with an exceptionally low or high price.

However, the total performance of fallen angel strategies does not exceedingly originate in American stocks. If we had performed the same strategies in an index with only the European and Asian constituents of the GPR 250, we would have obtained slightly lower, but similar out performances. Moreover, all other conclusions that are drawn from the volatility analysis below hold for a universe with only European and Asian constituents.

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[1] Traders who must trade a given number of shares in a certain period
[2] Traders who have liquidity demands that need not be satisfied immediately,

## *7.3 Volatility of Fallen Angel investment strategies*

A first, general assessment of the volatility of investment strategies is expressed as the standard deviation in the individual performances of the strategy. It shows that the volatility of the strategy is directly dependent on the volatility of the Index. Moreover, the volatility of the strategy goes hand in hand with its high dependence on a few peaks in individual performances.

### *7.3.1 Standard Deviation*

Huge swings in individual returns of an investment are evidently far from ideal. Preferably, the strategy would steadily outperform the market. Standard deviation is the statistical measurement that best sheds light on these historical fluctuations or volatility. It allows the strategy's performance swings to be captured into a single number. The standard deviation, or dispersion indicates how much the return of a strategy or fund is deviating from the expected normal returns. In case of a normal distribution, ceteris paribus, future returns of the strategy will be within one standard deviation 68% of the time and between two standard deviations 95% of the time. The standard deviations per strategy are obtained as follows.

$$\sigma = \sqrt{\frac{1}{K}\sum_{k=1}^{K}\left(pf_{fa,k} - \overline{pf_{fa}}\right)^2} \tag{14}$$

$\sigma$ Standard deviation in individual performances of strategy

$K$ Total transactions in complete period $= \frac{2715}{\text{h}} - 1$

$pf_{fa,k}$ Performance of fallen angels for transaction $k$

$\overline{pf_{fa}}$ Average individual performance of fallen angels

Number of Fallen Angels in Portfolio

| Holding period | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1 | 5.94 | 4.07 | 3.28 | 2.79 | 2.53 | 2.32 | 2.09 |
| 2 | 7.82 | 5.73 | 4.69 | 4.06 | 3.48 | 3.18 | 2.97 |
| 3 | 10.4 | 7.03 | 6.03 | 4.92 | 4.33 | 3.88 | 3.55 |
| 4 | 11.0 | 7.67 | 6.06 | 5.03 | 4.47 | 4.05 | 3.80 |
| 5 | 10.9 | 8.25 | 6.95 | 6.26 | 5.64 | 5.27 | 4.73 |
| 10 | 17.1 | 13.5 | 10.7 | 9.06 | 8.11 | 7.52 | 6.87 |
| 25 | 45.5 | 30.1 | 21.4 | 17.7 | 14.7 | 14.0 | 12.5 |
| 50 | 42.8 | 27.1 | 21.8 | 18.0 | 15.3 | 14.2 | 13.8 |
| 100 | 37.1 | 29.5 | 21.8 | 18.1 | 14.2 | 11.5 | 10.3 |

Figure 7.2: Table showing the standard deviations in percentages in the individual performances of fallen angel investment strategies.

As was hypothesized before, the standard deviation considerably decreases as *n* increases. On top, there seems to exist a positive correlation between holding period and standard deviation. To get an idea as to how large these standard deviations are, they should be seen in the light of average individual performances per trade. The table below shows the means of the list with individual performances per trade.

Number of Fallen Angels in Portfolio

| Holding period | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1 | 0.412 | 0.246 | 0.170 | 0.114 | 0.0699 | 0.0421 | 0.0353 |
| 2 | 0.444 | 0.293 | 0.273 | 0.227 | 0.196 | 0.151 | 0.117 |
| 3 | 1.31 | 0.744 | 0.687 | 0.576 | 0.496 | 0.467 | 0.460 |
| 4 | 0.942 | 0.971 | 0.723 | 0.547 | 0.540 | 0.473 | 0.430 |
| 5 | 1.02 | 0.956 | 0.914 | 0.955 | 0.891 | 0.818 | 0.730 |
| 10 | 2.45 | 2.07 | 1.54 | 1.33 | 1.13 | 0.976 | 0.982 |
| 25 | 5.30 | 4.63 | 3.46 | 3.13 | 2.52 | 2.54 | 2.20 |
| 50 | 7.22 | 6.69 | 4.19 | 3.58 | 1.59 | 1.40 | 1.28 |
| 100 | -0.606 | 3.28 | 0.113 | 1.32 | -0.374 | -0.611 | -0.186 |

Figure 7.3: Table showing the mean of the individual performances of fallen angel strategies in percentages.

Note that the standard deviations are huge relative to the individual performances. Standard Deviations of 10 times the average individual performance or higher are the rule rather than the exception. In other words, for a mean performance of, say, 0.4% for the 1-1 strategy, 32% of the individual performances are either lower than -5.6% or greater than 6.4%. As said, these fluctuations drop as you select a larger fallen angel portfolio.

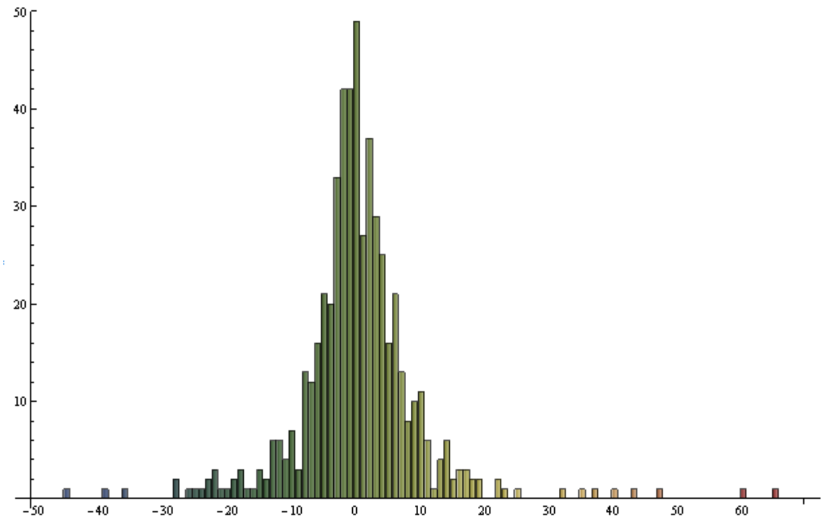


Figure 7.4: Chart showing the distribution of performances per trade for a 1/1 strategy: number of fallen angels is 1, holding period is 1 day.

*7.3.2 Variability in strategy versus the volatility of the Index*

The standard deviation of Fallen Angel strategies, the volatility of the strategy, appears to be greatly dependent on the volatility of the GPR 250 Index as a whole.

| | Std.Dev. Index | Std.Dev. Strategy |
|---|---|---|
| **2000** | 0.535 | 5.63 |
| **2001** | 0.539 | 5.49 |
| **2002** | 0.607 | 4.52 |
| **2003** | 0.493 | 2.71 |
| **2004** | 0.616 | 2.36 |
| **2005** | 0.542 | 2.52 |
| **2006** | 0.630 | 3.34 |
| **2007** | 1.09 | 5.01 |
| **2008** | 2.25 | 12.2 |
| **2009** | 1.93 | 8.68 |
| **2010** | 1.73 | 4.47 |
| **Correlation** | | 0.927 |

Figure 7.5: Correlation between volatility of the index and the risk of fallen angel strategies. Volatility of Index is given as the standard deviation in daily returns of the GPR 250 Index. Risk of the strategy is given as the standard deviation in performances of fallen angel strategies (*h* = 1 day, *n* = 1). The correlation gives the Pearson correlation coefficient between the volatility of index and that of the

The volatility of the strategy shows a great correlation with the volatility in the GPR 250 Index. From 2007 onwards, the latter volatility dramatically increases. This causes the graph of strategy through time to start jump up and down around the end of 2007. Beneath, some typical graphs of fallen angel strategies are displayed.

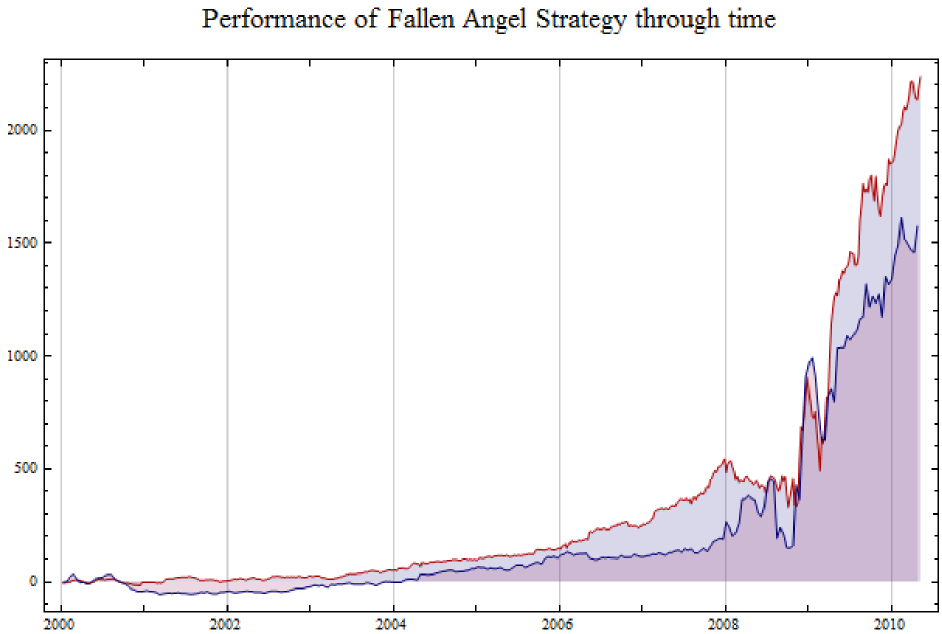


Figure 7.6: Two typical graphs of performance of fallen angel strategies through time. They steadily gain ground versus the Index, and start jumping up and down at the end of 2007, when the GPR 250 Index becomes more volatile. The strategies are 5/10 (red) and 10/3 (blue).

In the first 7 years of the dataset, the time in which the Index was not too variable, the performances of most fallen angel strategies are positive. Some are higher than the total performance in ten years. These performances are achieved with a considerably steadier pace, which suggests fallen angel investment strategies to perform very well in a stable property index.

Number of Fallen Angels in Portfolio

| Holding period | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1 | 114. | 31.6 | 13.7 | -1.30 | -5.57 | -7.85 | -11.2 |
| 2 | 24.2 | 9.94 | 14.6 | 14.2 | 14.7 | 10.5 | 10.7 |
| 3 | 22.8 | 21.8 | 22.3 | 19.2 | 23.7 | 22.0 | 21.2 |
| 4 | 66.0 | 57.3 | 40.5 | 31.0 | 23.1 | 18.1 | 16.7 |
| 5 | 16.0 | 32.8 | 30.8 | 34.6 | 31.3 | 31.3 | 23.3 |
| 10 | 0.800 | 6.59 | 12.1 | 14.6 | 14.3 | 14.7 | 12.1 |
| 25 | 13.7 | 18.9 | 14.4 | 14.8 | 15.4 | 14.0 | 12.1 |
| 50 | -12.8 | 13.0 | 9.10 | 5.75 | -1.20 | -0.801 | 0.143 |
| 100 | -22.0 | -11.1 | -10.2 | -2.21 | -2.79 | -1.26 | -0.293 |

Figure 7.7: Table showing the average annual performances of fallen angel strategies from 2000 until the end of 2006.

*7.4 Dependence on tails of individual performance distribution.*

The large deviations in the individual performances inevitably provoke the question as to what extent the total performances are caused by the few peaks in the individual performances. That is, by the tails of the individual performance distribution. We may separate the total list of individual performances in ten deciles with subsequent performances to get a better idea of the distribution.

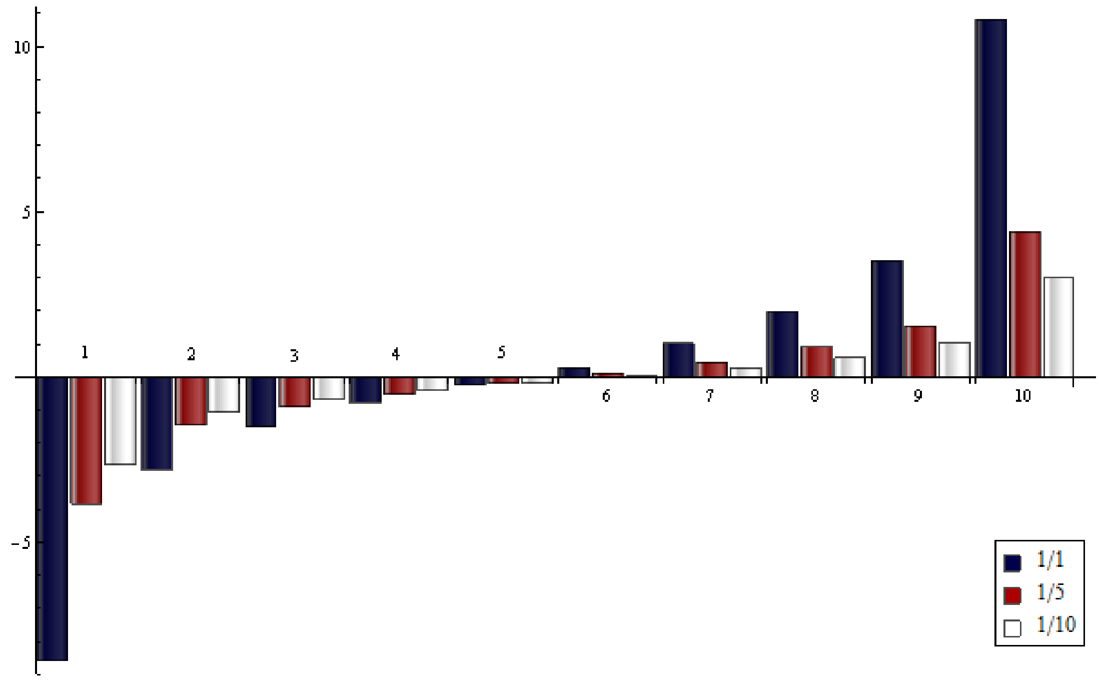


Figure 7.8: Average Return per trade in percentage for ten deciles with subsequent performances. Each decile contains an equal number of transactions. Three different strategies are pictured in this chart: 1/1 (blue), 5/1 (red), and 10/1 (white). Note how the outer deciles hugely deviate. In other words, the total return of the strategy is highly dependent on the deciles with worst and best performances.

The dependence on the few worst and best performances greatly differs for different fallen angel investment strategies. There are two trends clearly noticeable. Firstly, the importance of the outer bins decreases as the number of fallen angels in each portfolio increases. In other words, the strategy performance is decreasingly dependent on a few (un)lucky peaks as *n* increases.

| | ***1/1*** | ***1/5*** | ***1/10*** |
|---|---|---|---|
| ***Std. Dev*** | 4.94 | 2.13 | 1.47 |

Figure 7.9: Table showing the standard deviations in the deciled average performances for three Fallen Angel investment strategies with an increasing number of fallen angels in each portfolio. The smaller the standard deviation, the greater the coherence between these performances, and thus the smaller the role of the peaks.

Secondly, the role of the peaks increases as the holding period increases. In other words, the longer the holding period (the less trades per year), the greater the dependence of the total performance on a few (un)lucky peaks.

| | ***1/1*** | ***5/1*** | ***10/1*** |
|---|---|---|---|
| ***Std. Dev*** | 4.94 | 9.66 | 10.5 |

Figure 7.10: Table showing the standard deviations in the deciled average performances for three Fallen Angel investment strategies with an increasing amount of days in each holding period.

Still, in all positively performing Fallen Angel investment strategies, the role of the few (un)lucky peaks in individual performances is considerable. To find out how these peaks influence the total performance of the strategies, let us exclude the four most deviating bins. That is, we disregard the 20% worst performances and the 20% best performances. Figure 9.11 shows the resulting total performance after such exclusion.

| Holding Period | Number of Fallen Angels in Portfolio | | | | |
|---|---|---|---|---|---|
| | ***1*** | ***2*** | ***3*** | ***5*** | ***10*** |
| ***1*** | 20.4% (84.8%) | 16.3% (53.5%) | 6.18% (35.3%) | -4.27% (10.5%) | -9.96% (-2.57%) |
| ***2*** | 7.01% (17.5%) | 3.07% (18.5%) | 5.31% (24.3%) | 2.59% (19.7%) | 1.01% (12.3%) |
| ***5*** | 11.8% (28.2%) | 13.1% (39.3%) | 12.6% (42.9%) | 14.2% (46.8%) | 9.95% (35.4%) |
| ***10*** | 7.25% (38.6%) | 10.1% (41.2%) | 9.45% (31.1%) | 8.10% (24.6%) | 6.31% (22.6%) |

Figure 7.11: Table showing the annual performances of fallen angel strategies, disregarding the tails of the distribution of individual performances, the 20% worst and the 20% best performances. Between brackets is the total performance per strategy including the tails.

Apparently, the risky, unpredictable tails have considerably pushed the performances of fallen angel strategies in the positive direction over the past ten years. However, having cut off the tails, most strategies still show considerable outperformances. This table is one of the most important results of this research, as it suggests the existence of a stable profitable basis for some fallen angel strategies, independent on the very variable peaks in individual performances.

*7.5 Dependence on a few volatile stocks*

Besides a dependence on a few peaks in the individual performances, one may expect a high dependence of the total strategy on a small amount of volatile stocks that is continuously included in the fallen angel portfolios.

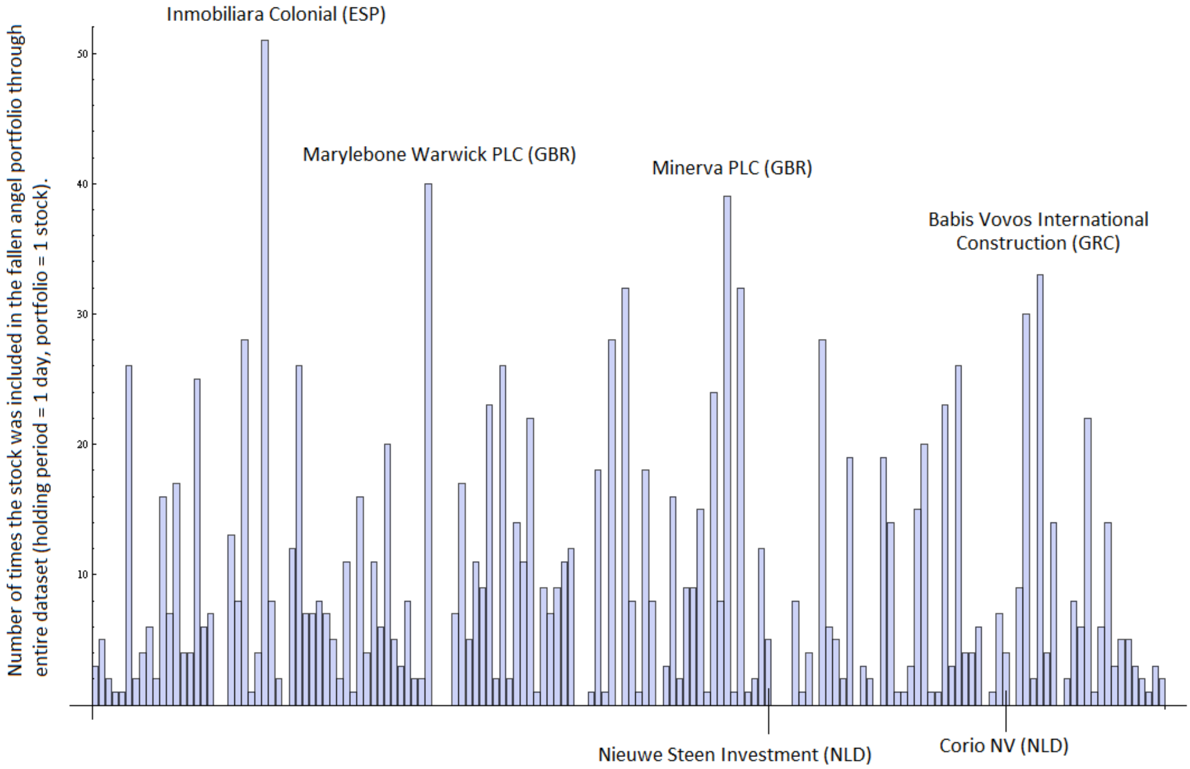


Figure 7.12: Chart showing the crooked distribution of abundance in Fallen Angel portfolios among European constituents of the GPR 250 for a 1/1 strategy. Clearly, some companies are included in fallen angel portfolios a great deal more often than others.

Naturally, the question arises to what extent the total performance is caused by a few most abundant stocks. The results show that such a premise is out of the question.

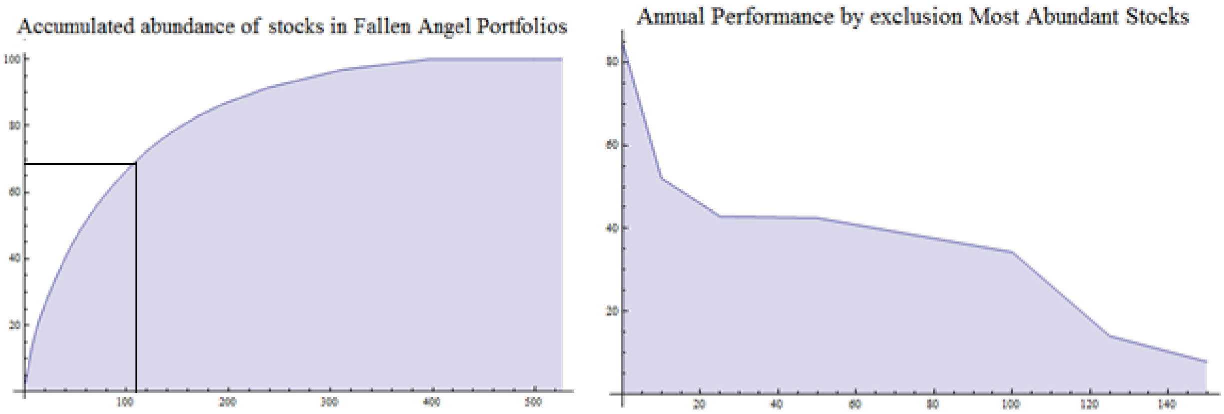


Figure 7.13: Left: Graph of the accumulated abundance of stocks in Fallen Angel portfolios for 1/1 strategy in percentages of total abundance. On the x-axis, the number of stocks sorted by abundance. Note that 20% of the stocks take up over half of the portfolios. Total number of stocks is 529. Right: Annual Performance of strategy 1/1 by exclusion most abundant stocks. On the x-axis the number of most abundant stocks that is excluded from selection in fallen angel portfolios; they are replaced by the next fallen angels. On the y-axis the resulting average annual performance in percentages.

From the shape of left graph, we conclude that the distribution of stocks included in the fallen angel portfolios is indeed very skewed; that is, some stocks are included very often, whereas other are never included (if every stock would be included equally much, the line would approximate a straight diagonal). However, from the right graph, the total performance strategy is not disproportionally dependent on the most abundant stocks. The greater the downward slope, the greater the performance of the relating stocks. Although the ten most abundant stocks do add somewhat more to the performance than others, the trend line overall is still gradually downward sloping, which suggests that every group of stocks adds their fair share to the total performance.

## 8.0 Conclusion

This research study shows the great potential fallen angel investment strategies have in correlated markets. With average annual performances of up to 100% in the ten year period, fallen angel strategies prove to be a greatly profitable investment strategy in recent history. However, we have to make a few annotations to that outcome before we project the historic performance on the future. Overall, the volatility throughout the fallen angel strategies is hefty, which enlarges the risk of the strategy. The standard deviation of the strategy is greatly correlated with the volatility of the index. In volatile times of the GPR 250 Index, huge plummets in de performance of the strategy alternated huge peaks. In other words, the risk of a fallen angel strategy reflects the volatility of the Index in which you perform the strategy.

Although the total performances of most strategies chiefly originates in the volatile period after 2007, these results cannot be readily projected on the future. During stable times in the property sector, from 2000 until 2007, almost all strategies with small portfolios and short holding periods gradually outperformed the market. Seemingly it is in these stable times of the property sector, that fallen angel strategies are especially profitable and stable.

There are some other arguments to be made that suggest a stable basis of profitability for fallen angel strategies in the property sector. Throughout the entire period, the total performances are greatly dependent on the peaks in individual performances. However, the body of the performances, the 60% individual performances that are closest to the mean, still generates a considerable positive performance for most strategies. This suggests a stable basis for fallen angel strategies that, independently on the few lucky peaks, generates an outperformance with respect to the index. Moreover, the strategy is not disproportionally dependent on a few very volatile, risky stocks.

All in all, fallen angel investment strategies in the GPR 250 Index have performed exceptionally well in the past ten years. In times of stable Index, most all fallen angel strategies gradually outperform the market, and throughout the entire period, the 60% individual performances that are closest to the mean generate a considerable outperformance.

Thus, to end along the words of Mark Cuban, it seems that "if you are not performing fallen angel investment strategies in the property sector, you are missing some amazing outperformance".

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